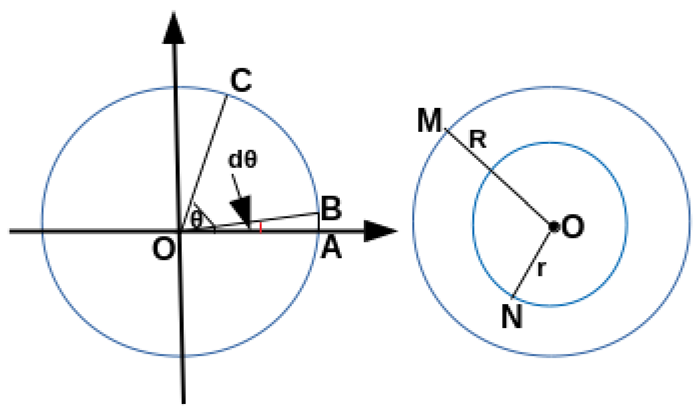
Derivation Of Area Of Circle, Sector Of A Circle And Circular Ring

Area Of A Circle, Sector Of A Circle and Circular Ring



(a) circle (b) circular ring

Figure 113.2

Consider figure 113.2 (a)

Area of Circle = πr^2

Area of Sector Of A Circle = $(1/2)r^2\theta$

<u>Derivation of Area of Circle</u>

Line OA = radius of circle = r
Line OB = radius of circle = r
Let angle, < BOA = dθ radians
Let line BA be perpendicular to line OA at A
Then BA = r sin dθ
Let the area of Triangle OAB = dA
Then dA = (½)(r)(rsin dθ) = $(r^2/2)$ sin dθ
So, dA/dθ = $(r^2/2)$ (sin dθ)/dθ
Limit $dA/d\theta_{\ d\theta \to 0}$ = limit $r^2/2_{\ d\theta \to 0}$ limit $(\sin d\theta)/d\theta_{\ d\theta \to 0}$
Limit $(\sin d\theta)/d\theta_{\ d\theta \to 0}$ = 1
So, dA/dθ = $r^2/2$
So, dA = $(r^2/2)d\theta$

So from integral calculus, area of circle $= \int_0^{2\pi} dA = \int_0^{2\pi} (r^2/2)\, d\theta = \pi r^2$ --------------(1)

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area

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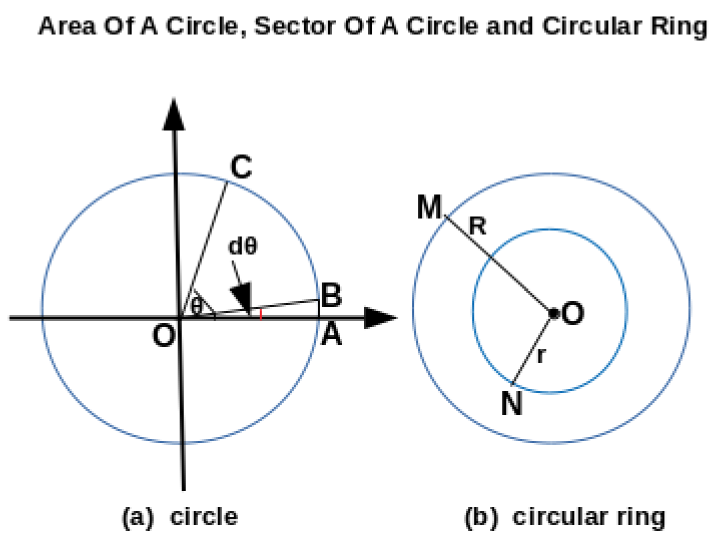

Figure 113.2

Derivation of Area of Sector of Circle

Consider sector COA of circle

So ,from integral calculus, area of sector COA = $\int_0^{\theta} \mathbf{dA} = \int_0^{\theta} (\mathbf{r^2/2})\,\mathbf{d\theta} = \mathbf{(r^2/2)\theta}$ --------------(2)

Derivation of Area of Circular Ring

Consider figure 113.2 (b). Area of circular ring is area of outer circle with radius R minus area of inner circle with radius r.

Area of outer circle = πR^2
Area of inner circle = πr^2

So, Area of circular ring = $\pi R^2 - \pi r^2$

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area

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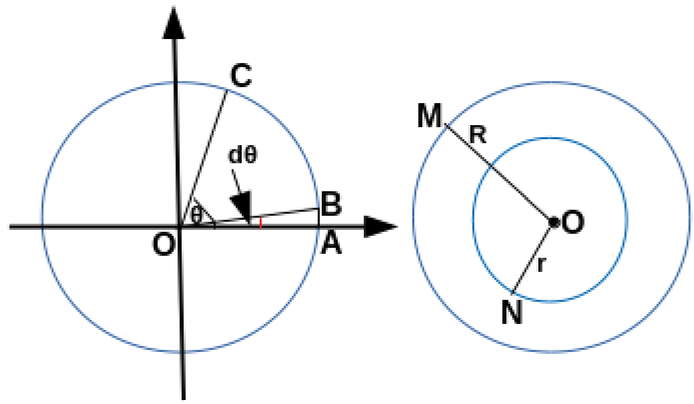

(a) circle (b) circular ring

Figure 113.2

Alternate Derivation of Area of Circle

Consider first quadrant of circle (figure 113.2 (a)).
equation of circle with center at origin and radius r is $\mathbf{x^2 + y^2 = r^2}$

So, $\mathbf{x = \sqrt{(r^2 - y^2)}}$

Let y = rsinθ
Then dy/dθ = rcosθ
So, dy = rcosθdθ
When y = 0, sinθ = 0. When y = r, sinθ = π/2

So, $\mathbf{x = \sqrt{(r^2 - y^2)} = \sqrt{(r^2 - r^2\sin^2\theta)} = r\cos\theta}$

So, area under curve in first quadrant $= \int_0^{\pi/2} (\mathbf{r^2\cos^2\theta})\,\mathbf{d\theta} = \mathbf{r^2\int_0^{\pi/2} (1/2)(1+\cos 2\theta)\,d\theta}$

$$= \mathbf{r^2/2\int_0^{\pi/2} (1+\cos 2\theta)\,d\theta = r^2/2[\theta + (1/2)\sin 2\theta]_0^{\pi/2}}$$
$$= \mathbf{\pi r^2/4}$$

So, area of circle = 4 x area under curve in first quadrant = $\mathbf{\pi r^2}$

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area

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