Derivation Of Area Of A Trapezoid, A Rectangle And A Triangle

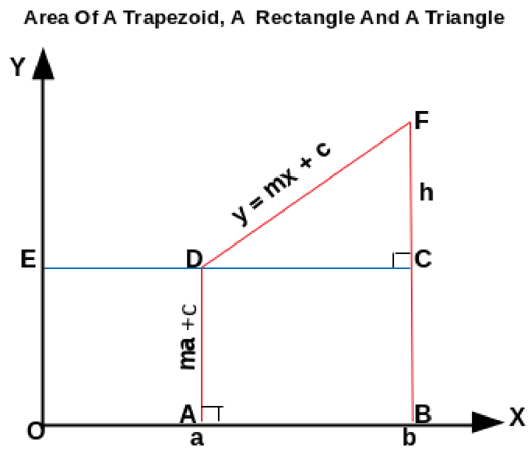

Figure 113.1

Consider figure 113.1. ABFD is a trapezoid, ABCD is a rectangle and DCF is a triangle.

Area of ABFD = (1/2)(b-a)[ma +c + ma + c + h] = (1/2)(b-a)[2ma + 2c + h]

Area of ABCD = (b-a)(ma + c)

Area of DCF = (1/2)(b-a)h

<u>Derivation of Area of Trapezoid, ABFD</u>

Equation of line DF: **y = mx + c**
Area of ABFD = Area enclosed by line DF, line x =a, line x = b and line y = 0.

So from integral calculus, area of ABFD = $\int_a^b mx + c\, dx$ --------------------(1)

$= [(mb^2/2) + cb] - [(ma^2/2) + ca]$

$= m(b^2 - a^2)/2 + c(b-a)$

= **[(b-a)/2] [m(a + b) + 2c], where m = h/(b-a).**

So, **[(b-a)/2] [m(a + b) + 2c] = [(b-a)/2] [(h(a + b)/(b-a) + 2c]**

= **(1/2)(b-a)[2ma + 2c + h] when m = h/(b-a).**

Area of ABFD = (1/2)(sum of lengths of parallel sides)(perpendicular distance between parallel sides)

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area

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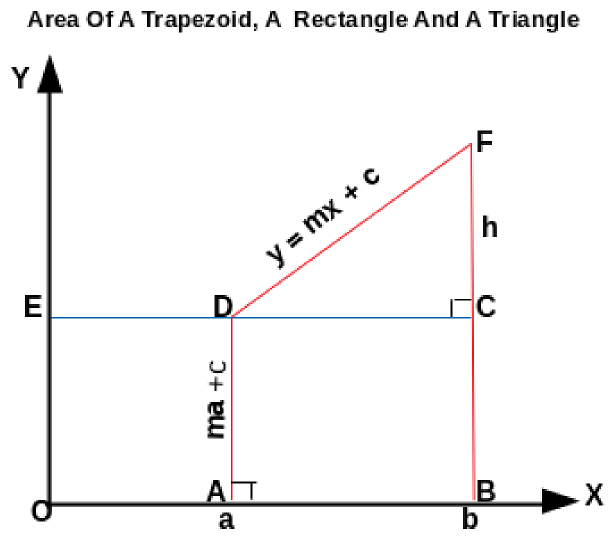

Figure 113.1

Derivation Of Area of Rectangle, ABCD

Equation of line DC **y = ma + c**
Area of ABCD = Area enclosed by line DC, line x =a and line x = b and line y = 0.

So from integral calculus, area of ABCD = $\int_a^b$ **ma + c dx** --------------------(2)

$= (mab + cb) - (ma^2 + ca)$

$= m(ab - a^2) + c(b - a)$

$= ma(b - a) + c(b - a)$

$= (b - a)(ma + c).$

So, Area of rectangle = (length of rectangle)(breadth of rectangle)

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area

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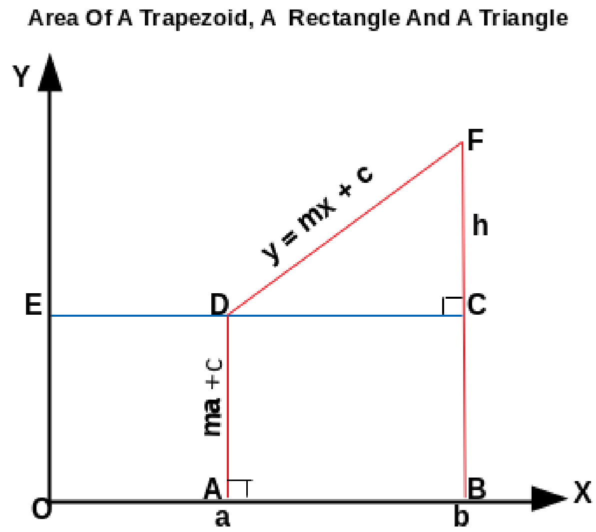

Figure 113.1

Derivation of Area of Triangle DCF

Area of triangle DCF = Area of trapezoid ABFD – Area of rectangle ABCD.

So, from integral calculus area DCF = $\int_a^b$ **mx + c dx** - $\int_a^b$ **ma + c dx** ----------------(3)

$= [(mb^2/2) + cb] - [(ma^2/2) + ca] - [(mab + cb) - (ma^2 + ca)]$

$= mb^2/2 - ma^2/2 - mab + ma^2$

$= ((b - a)/2)m[b + a - 2a]$

$= ((b - a)/2)m(b-a) = ((b - a)/2)\, h$, where $m = h/(b-a)$.

So, area of triangle = (1/2)(base of triangle)(height of triangle)

The string is $S_1P_1A_{13}$ - Empty Space -Containership - Area