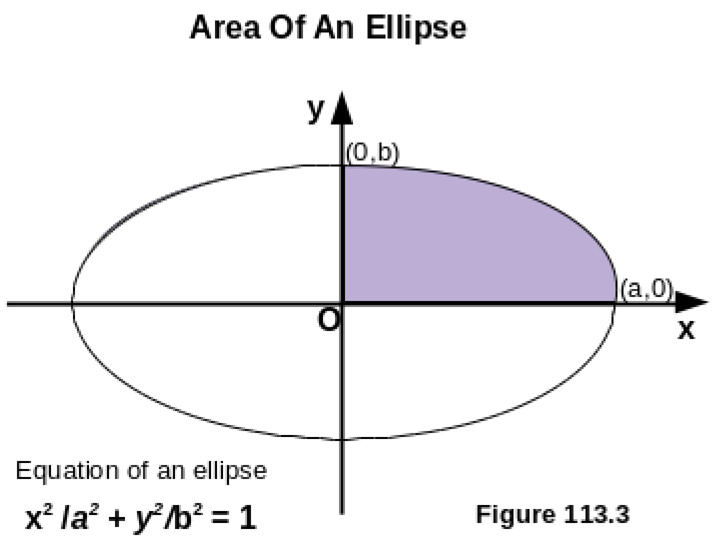
# Derivation Of Area Of An Ellipse




A standard ellipse is illustrated in figure 113.3. The origin of the y and x axes is at its center, the point where its major axis (x-axis) and its minor axis (y axis) intersect.

Equation of ellipse is $\mathbf{x^2/a^2 + y^2/b^2 = 1}$
Area of ellipse $= \boldsymbol{\pi ab}$

**<u>Derivation of Area of an Ellipse</u>**

From equation of ellipse, $y^2 = b^2(1 - x^2/a^2) = (b^2/a^2)(a^2 - x^2)$

So, $y = \pm (b/a)\sqrt{(a^2 - x^2)}$

Curve of ellipse in first quadrant is $y = (b/a)\sqrt{(a^2 - x^2)} \quad 0 \le x \le a$

So, area of first quadrant $= \int_0^a (b/a)\sqrt{(a^2 - x^2)}dx$ --------------(1)

integral evaluation by substitution

let $x = a\sin\theta$. Then $dx = a\cos\theta d\theta$

When x = 0, θ = 0. When x = a, θ = π/2

So, equation (1) $= \int_0^{\pi/2} (b/a)\sqrt{(a^2 - a^2\sin^2\theta)}\, a\cos\theta d\theta$ Since x = asinθ,

$= (b/a)\int_0^{\pi/2} \sqrt{(a^2\cos^2\theta)}\, a\cos\theta d\theta$

# Derivation Of Area Of An Ellipse

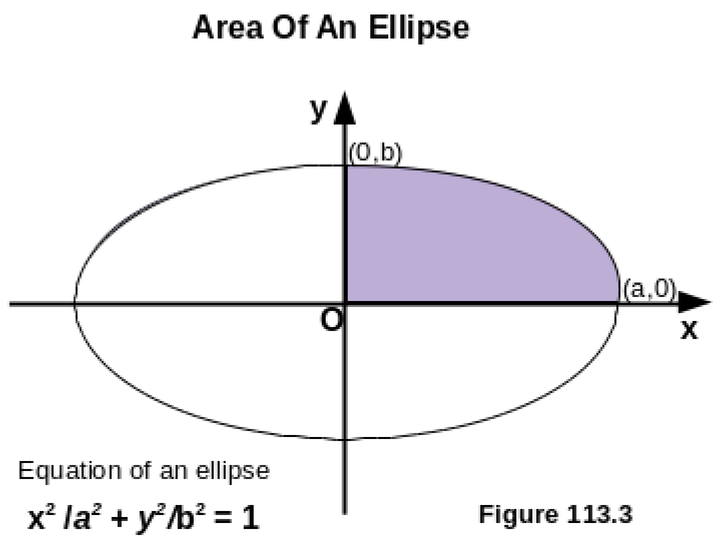


Figure 113.3

So, $$\mathbf{(b/a)}\int_0^{\pi/2} \sqrt{(a^2\cos^2\theta)}\, a\cos\theta\, d\theta = ab\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= ab\int_0^{\pi/2} (1/2)(1 + \cos 2\theta)\, d\theta$$

$$= (ab)/2\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= (ab)/2\,[(\pi/2 + (1/2)\sin 2\pi/2) - (0 + 0)]$$

$$= (\pi/4)ab$$

Since the ellipse is symmetric with respect to both its axes, its area is 4 times the area of its first quadrant.

So, area of ellipse = **4(π/4)ab = πab.**

**The string is $S_1P_1A_{13}$ - Empty Space – Containership - Area**