Advanced Calculus – Multiple Integrals

/ is the division symbol

Definitions
Domain two types: *open domain (*D_o) – an open connected set of points; *bounded domain (*D_b) – all points of domain lie inside some square.

Region (R): *open region* = open domain; *closed region* = domain + all its boundary; *partially bounded* region = domain + some of its boundary.

Region $\mathbf{R_x = [a, b, f(x), g(x)]}$ where f(x), g(x) Є C in $a \le x \le b$ and $f(x) < g(x)$ in $a \le x \le b$ implies R_x is the region bounded by x = a, x = b, y = f(x), y = g(x).

(1) Describe the region:

$$R_y = [-4, 5, -\sqrt{(25 - y^2)}, 3y/4]$$

Express it as the sum of several regions R_x

Ans (1)

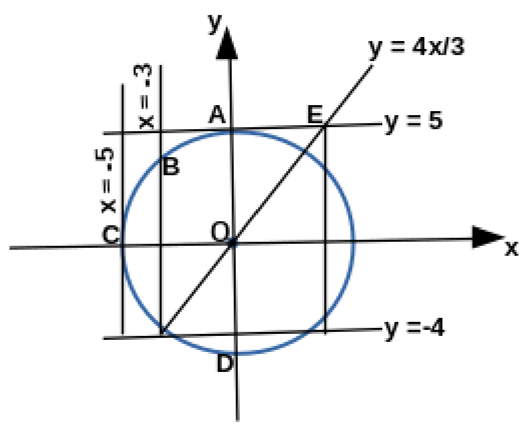

Figure 9.1

Consider figure 9.1, Only the semi-circle ABCD is under consideration not the entire circle.
$R_x = R_1 + R_2 + R_3$.
$R_1 = [-5, -3, -\sqrt{(25 - x^2)}, \sqrt{(25 - x^2)}]$
$R_2 = [-3, 0, 4x/3, \sqrt{(25 - x^2)}]$
$R_3 = [0, 15/4, 4x/3, 5]$

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(2) Decompose the set of points (x, y) for which $2 \le x^2 + y^2 \le 4$ into two regions R_x. A point of the set may be a boundary point of both region R_x.

Ans (2) The region R of interest is the region shaded blue in figure 9.2.

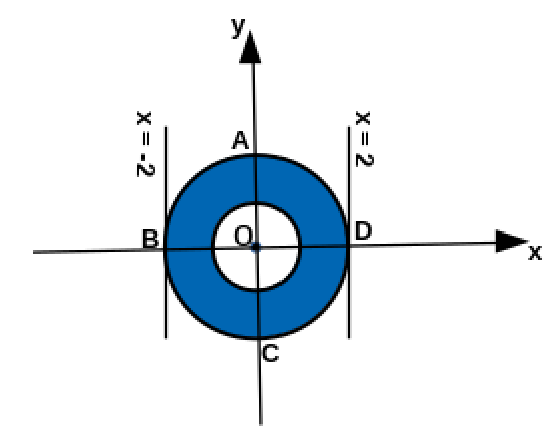

Figure 9.2

$R_x = R_1 + R_2$. R_1 is the region with the outer boundary BAD; R_2 is the region with outer boundary BCD.
$R_1 = [-2, 2, g(x), \sqrt{4 - x^2}]$
$R_2 = [-2, 2, -\sqrt{(4 - x^2)}, -g(x)]$
Where $g(x) = \sqrt{(2 - x^2)}, x^2 < 2$; $g(x) = 0, x^2 \ge 2$.

Definitions

Δ (delta): subdivision of a region R. Established by a set of closed curves $\{C_k\}^n_1$ which divides R into n subregions R_k of area ΔS_k, k = 1, 2, 3, …, n.

$\|\Delta\|$ (norm of a subdivision Δ): the maximum diameter of the subregions produced by the subdivision.

Double integral of f(x, y) over the region R:

$$\int\int_R f(x, y)\, dS = \lim_{\|\Delta\| \to 0} \sum_{k=1}^{n} f(\xi_k, \eta_k)\, \Delta S_k .$$ Where (ξ_k, η_k) is a point of R_k.

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Iterated Integrals: integrals of the form:

$$\int_b^a dx \int_{g(x)}^{h(x)} f(x, y)dy$$

Volume Of A solid:

$$V = \int_R \int f(x, y)\, dS$$

Fundamental Theorem Of Double Integral:

$f(x, y) \in C$ in R_x,
$R_x = [a, b, g(x), h(x)]$

implies: $\int_R \int f(x, y)\, dS = \int_b^a dx \int_{g(x)}^{h(x)} f(x, y)dy$ --------------(1)

(3) Evaluate:

$$\int_0^{\pi/4} dx \int_0^{\sec x} y^2\, dy$$

Ans(3) $\int_0^{\pi/4} 1/3 \sec^3 x\, dx = (1/3)[(1/2)(\sec x\tan x + \ln(\sec x + \tan x))]_0^{\pi/4} = (1/3)(1) = 1/3.$

Change In Order Of Integration
If $f(x, y) \in C$ in a suitable region R, then:

$$\int_a^b dx \int_a^x f(x,y)\, dy = \int_a^b dy \int_y^b f(x,y)\, dx$$ (**Dirichlet's formula**)

(4) Interchange the order of integration:

$$\int_0^1 dx \int_{x^2}^1 f(x,y)\, dy$$

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Ans(4)

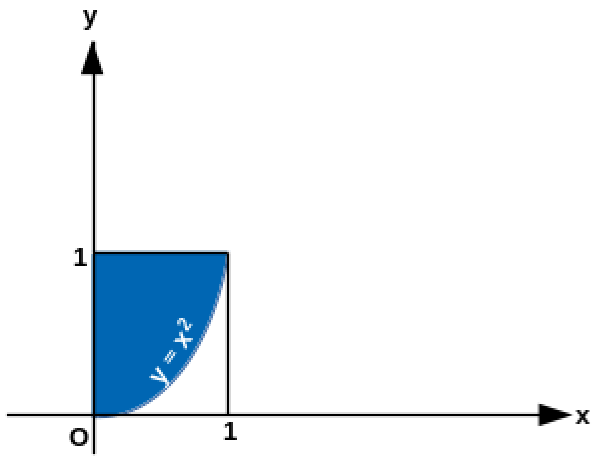

Figure 9.3

Consider figure 9.3, $R_y = [0, 1, 0, \sqrt{y}]$, so by Dirichlet's formula:

$$\int_0^1 dx \int_{x^2}^1 f(x,y)\, dy = \int_0^1 dy \int_0^{\sqrt{y}} f(x,y)\, dx$$

Triple Integrals: integrals of functions of three variables over regions of three dimensional space. Fundamental evaluation similar to fundamental theorem of double integrals (1)

$$\int\int\int_{V_{xy}} f(x, y, z)\, dV = \int\int_R dS \int_{g(x,y)}^{h(x,y)} f(x, y, z)\, dz \text{ ----------------(2)}$$

Peter Oye Simate Sagay
Simate was my mother
Sagay was my father