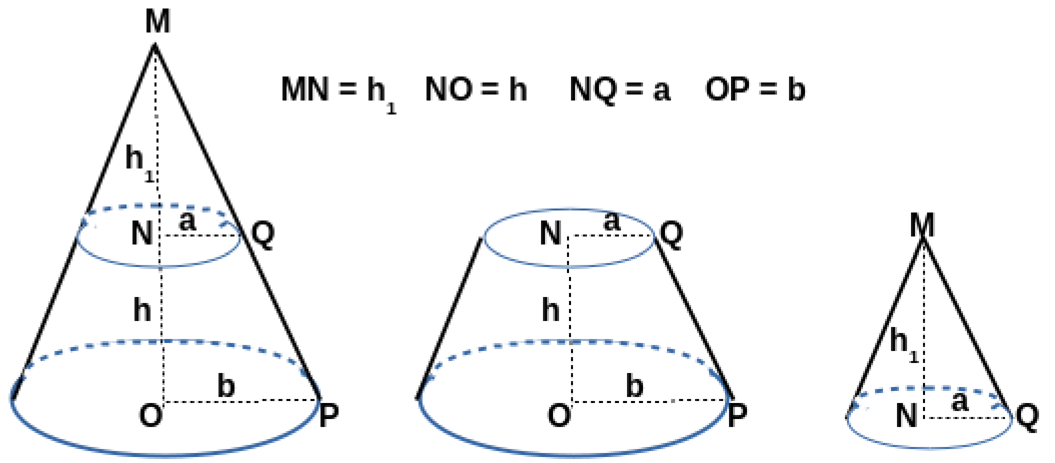
# Derivation Of Volume Of A Frustum

**Volume Of A Frustum** **Figure 114.3**




(i) Large cone (ii) Frustum (iii) Small cone

When a horizontal plane slices through the cone in figure 114.3(i) at Q, the shape below the plane is a frustum and the shape above the plane is a smaller cone.
Volume of frustum = $\mathbf{(1/3)\pi h(a^2 + ab + b^2)}$

**<u>Derivation Of Volume Of A Frustum</u>**

Volume of frustum = **volume of large cone – volume of small cone**
= $\mathbf{(1/3)\pi(b^2)(h_1 + h) - (1/3)\pi(a^2)(h_1)}$ ----------------------(1)

Triangle MNQ is similar to Triangle MOP. N, and O are centers of circles Q and P respectively.
So, $\mathbf{h_1/(h_1 + h) = a/b}$ ----------------------(2)
So, $\mathbf{h_1 = ha/(b-a)}$ -----------------------(3)

Replacing (**$(h_1 + h)$** in equation (1) with **$(h_1 b)/a$** we have:

$$\mathbf{(1/3)\pi(b^2)(h_1 + h) - (1/3)\pi(a^2)(h_1) = (1/3)\pi[b^2(h_1 b)/a - (a^2)(h_1)]}$$

$$\mathbf{= (1/3)\pi[(h_1/a)(b^3 - a^3)]}$$

But **$h_1/a = h/(b-a)$** from equation (3)
So, $\mathbf{(1/3)\pi[(h_1/a)(b^3 - a^3)] = (1/3)\pi h[(b^3 - a^3)/(b-a)] = (1/3)\pi h(a^2 + ab + b^2)}$

So, volume of frustum = $\mathbf{(1/3)\pi h(a^2 + ab + b^2)}$

**The string is $S_1P_1A_{14}$ - Empty Space – Containership - Volume**

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