## Derivation Of Volume Of A Sphere

## Volume Of A Sphere



Figure 113.9
The sphere illustrated in figure 113.9 is generated by revolving the circle with center at the origin and radius $r$ about the $y$ axis.
Volume of sphere $=(4 / 3) \pi \mathbf{r}^{3}$

## Derivation Of Volume Of A Sphere

Area of circular cross-section $=\pi \mathbf{x}^{2}$. Since radius of cross-section is x
Volume of cross-sectional area $=\pi\left(x^{2}\right) \mathbf{d y}$
Volume of sphere is twice volume of hemisphere

$$
\begin{align*}
\text { So, volume of sphere } & =2 \int_{0}^{\mathbf{r}} \pi\left(\mathbf{x}^{2}\right) \mathbf{d y}-\ldots  \tag{1}\\
& =2 \pi \int_{0}^{\mathbf{r}}\left(\mathbf{r}^{2}-\mathbf{y}^{2}\right) \mathbf{d y} \\
& =2 \pi\left(\mathbf{r}^{3}-\mathbf{r}^{3} / 3\right) \\
& =2 \pi\left(3 \mathbf{r}^{3}-\mathbf{r}^{3}\right) / 3 \\
& =(4 / 3) \pi \mathbf{r}^{3} .
\end{align*}
$$

The string is $\mathrm{S}_{1} \mathbf{P}_{1} \mathrm{~A}_{14}$ - Empty Space - Containership - Volume

