Derivation Of Volume Of A Torus

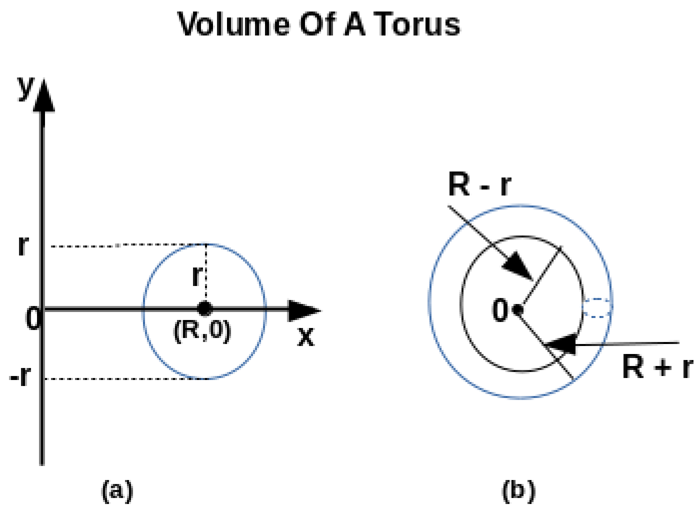

Figure 114.6

The torus in figure 114.6b is obtained by revolving the circle $\mathbf{(x – R)^2 + y^2 = r^2}$ in figure 114.6a about the y axis.

Derivation Of Volume Of A Torus

Equation of circle with center at (R,0) and radius r is $\mathbf{(x – R)^2 + y^2 = r^2}$
So at right half of circle (114.6a), $\mathbf{x = R + \sqrt{(r^2 – y^2)} = f(y)}$
So at left half of circle (114.6a), $\mathbf{x = R - \sqrt{(r^2 - y^2)} = g(y)}$

$$\text{So, volume of torus} = \int_{-r}^{r} \pi \left[(f(y))^2 - (g(y))^2\right]dy \text{-------------(1)}$$

$$= 2\pi\int_0^r 4R\sqrt{(r^2 - y^2)}dy$$

$$= 8\pi R\int_0^r \sqrt{(r^2 – y^2)}dy$$

$$= 8\pi R(\pi r^2/4) \quad \text{since} \int_0^r \sqrt{(r^2 – y^2)}dy = \text{area of a quarter of a circle}$$

$$\text{So, volume of torus} = 2\pi^2 r^2 R$$

The string is $S_1P_1A_{14}$ - Empty Space – Containership - Volume