Advanced Calculus – Fourier Series

/ is the division symbol

Fourier Series (named after mathematician Jean Baptiste Joseph Fourier) is a summation of trigonometric functions. The basic idea is to replace a function f(x) with the summation of trigonometric functions. This replacement is only possible if the coefficients of the trigonometric functions can be determined as required. Periodic functions are particular replacable by Fourier Series.

The summation Of Trigonometric functions:

Fourier Series: $(a_0)/2 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$ ----------------------------(1) k = 0, 1, 2, 3, ...

Where the coefficients, $a_k = (1/\pi)\int_{-\pi}^{\pi} f(x) \cos (kx)\, dx$ --(2)

And the coefficients, $b_k = (1/\pi)\int_{-\pi}^{\pi} f(x) \sin (kx)\, dx$ --(3)

(1) If $f(x) = x^2$, find the Fourier series for f(x). $-\pi < x < \pi$

Ans (1) $a_0 = (1/\pi)\int_{-\pi}^{\pi} x^2\, dx = (1/\pi)(\pi^3 + \pi^3)/3 = (2\pi^2)/3$

$b_k = (1/\pi)\int_{-\pi}^{\pi} x^2 \sin (kx)\, dx = 1/\pi[x^2(-\cos kx)/k - \int 2x(-\cos kx)/k\, dx]_{-\pi}^{\pi} = 0$ (integration by parts)

$a_k = (1/\pi)\int_{-\pi}^{\pi} x^2 \cos (kx)\, dx = (-1)^k 4k^{-2}$ for k = 1, 2, 3, … (integration by parts)

Integration by parts:
Functions, f(x), g(x); f' = derivative of f(x); G(x) = antiderivative of g(x)

$\int f(x)g(x)\, dx = f(x)G(x) - \int f'(x)G(x)\, dx$ (integration by parts).

Peter Oye Simate Sagay
Simate was my mother
Sagay was my father