

Advanced Calculus – General Characteristics Of Partial Differential Equations

Order, linearity, nonlinearity. Homogeneity, nonhomogeneity, parabolic, hyperbolic, elliptic

/ is the division symbol

For each of the partial differential equation (PDE) presented, determine the order, linearity or nonlinearity, homogeneity or nonhomogeneity and whether it is parabolic or hyperbolic or elliptic .

(1) $u_t = u_{xx}$

Ans (1) Order is 2; linear (power of depended variable < 2 , or and dependent variable not multiplied by its derivative); nonhomogeneous because cannot be expressed in the form:

$Au_{xx} + Bu_{xt} + Cu_{tt} + Du_x + Eu_t + Fu(x,t) = G(x,t)$ -----(1), such that $G(x,t) = 0$.

A partial differential equation is linear and homogeneous if it is expressed in the form of equation (1) and the G (right hand side) is zero. $A = 1, B = 0, C = 0$. Parabolic, $B^2 - 4AC = 0$.

(2) $u_t = uu_{xxx} + \sin x$

Ans (2) Order is 3; nonlinear (depended variable u multiplies its derivative)
Nonhomogeneous. Parabolic, hyperbolic and elliptic apply to linear PDEs.

(3) $u_t = u_{rr} + (1/r)u_r + (1/r^2)u_{\theta\theta}$

Ans (3) Order is 3; linear; however, not in the form of equation (1); nonhomogeneous.

(4) $u_{tt} = e^{-t}u_{xx} + \sin t$

Ans (4) Order is 2; linear; nonhomogeneous; $B = 0, A = -e^{-t}, C = 1$. Hyperbolic, $B^2 - 4AC > 0$.

(5) $uu_{xx} + u_t = 0$

Ans (5) Order is 2; nonlinear (dependent variable multiplies its derivative); nonhomogeneous.

(6) $u_{xx} + yu_{yy} = 0$

Ans (6) Order is 2; linear (y is an independent variable); $A = 1, B = 0, C = y$.

Elliptic, $B^2 - 4AC = -4y < 0$ (for $y > 0$)

Parabolic, $B^2 - 4AC = 0$ (for $y = 0$)

hyperbolic, $B^2 - 4AC > 0$ (for $y < 0$)

(7) $u_{xy} = 0$

Ans (7) Order is 2; linear; homogeneous, $B = 1, A = 0, C = 0$

Hyperbolic, $B^2 - 4AC = 1 > 0$.

Advanced Calculus – General Characteristics Of Partial Differential Equations

Order, linearity, nonlinearity. Homogeneity, nonhomogeneity, parabolic, hyperbolic, elliptic

(8) $u_{tt} = u_{xx}$

Ans (8) Order is 2; linear, nonhomogeneous; $A = -1, B = 0, C = 1$
Hyperbolic, $B^2 - 4AC > 0$.

(9) $u_{xx} + u_{yy} = 0$

Ans (9) Order is 2; linear; homogeneous; $A = 1, B = 0, C = 1$
Elliptic, $B^2 - 4AC < 0$.

(10) $yu_{xx} + u_{yy} = 0$

Ans (10) order is 2; linear; homogeneous; $A = y, B = 0, C = 1$
 $B^2 - 4AC = -4y$. Parabolic for $y = 0$; hyperbolic for $y < 0$; Elliptic for $y > 0$.

(11) $u_t = u_{xx} + 2u_x + u$

Ans (11) order is 2; linear, nonhomogeneous; $A = 1, B = 0, C = 0$
Parabolic, $B^2 - 4AC = 0$.

(12) $u_t = u_{xx} + e^{-t}$

Ans (12) Order is 2; linear, nonhomogeneous; $A = -1, B = 0, C = 0$
Hyperbolic $B^2 - 4AC > 0$.

(13) $u_{xx} + 3u_{xy} + u_{yy} = \sin x$

Ans (13) Order is 2; linear; nonhomogeneous; $A = 1, B = 3, C = 1$
Hyperbolic, $B^2 - 4AC > 0$.

(14) $u_{tt} = uu_{xxxx} + e^{-t}$

Ans (14) Order is 4; nonlinear (dependent variable multiplies its derivative).

Peter Oye Simate Sagay
Simate was my mother
Sagay was my father.