Production Schedule That Maximizes Profit Function Given Constraint Equation

Problem: A company makes two products, A and B. The products use the same raw materials. The company must decide on how to distribute its resources of fixed raw materials and fixed number of employees in the production of A and B. The *production possibilities equation* is as follows:

$$9x^2 + 4y^2 = 18000 \text{----------------------}(1)$$

Where x is the units of A produced and y is the units of B produced. The ordered pair (x,y) represents a *production schedule* for the company. The *production schedule* commits the company to produce x units of A and y units of B because of the constraint on raw materials and manpower.
The profit function when the company makes products A and B is as follows:

$$P(x,y) = 3x + 4y \text{ ----------------------}(2)$$

Where the coefficient of x, 3 is the company's profit for each unit of A; and the coefficient of y, 4 is the company's profit for each unit of B.

Determine the *production schedule* that maximizes the profit function P(x,y). In other words, determine the ordered pair (x,y) that maximizes P(x,y).

Solution: The method of partial differentiation and the Langrange Multiplier is suitable for this problem. PjProblem String: $S_7P_5A_{51}$ (change - physical)
Equation (1) is the constraint, so rearrange it and multiply it by the Langrange multiplier λ as follows:

$$18000 - 9x^2 - 4y^2 = 0$$

$$\lambda(18000 - 9x^2 - 4y^2) = 0$$

Form the function **F(x, y, λ) = 3x + 4y + λ(18000 - 9x² – 4y²)**
Find the partial derivatives of **F** with respect to x, y and λ and set them to zero as follows:

$$\delta F/\delta x = 3 + \lambda 18x = 0 \text{ ----------------}(3)$$

$$\delta F/\delta y = 4 + \lambda 8y = 0 \text{----------------}(4)$$

$$\delta F/\delta\lambda = 18000 - 9x^2 - 4y^2 = 0 \text{-------}(5)$$

From (3), $\lambda = -3/18x = -1/6x$
From (4), $\lambda = -4/8y = -1/2y$
So, $-1/6x = -1/2y$. So, $x = y/3$

Substitute x value into equation (5) and solve for y
So, $18000 - 9(y/3)^2 - 4y^2 = 0$
So, **y = 60 and x = 20**

So, company must make 20 units of product A and 60 units of product B to maximize profit.