Spectrum Of A Function

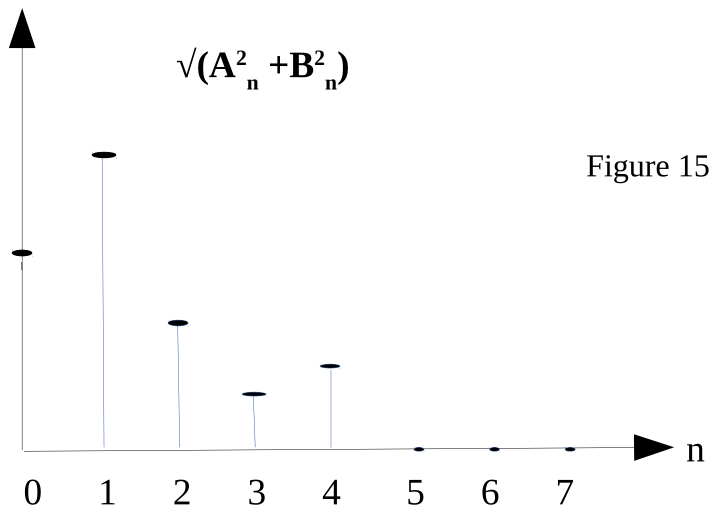

Figure 15.1

Figure 15.1 is a discrete spectrum of the periodic function f(x). Its Fourier series is:

$$\mathbf{f(x) = \sum_{n=0}^{\infty} [A_n \cos (nx) + B_n \sin (nx)]}$$

Suppose f(x) is resolved into a simple sum of sines and cosines as follows:

$$\mathbf{f(x) = 1 + \sin x + (1/5) \sin (3x) + \cos x + (1/2)\cos (2x) + (1/4)\cos (4x)}$$

Determine the magnitude of the components in f(x) with frequency n for n = 0, n = 1, n = 2 , n = 3 and n = 4 .

The coefficient A_n represents the amount of f(x) made up by cos (nx)
The coefficient B_n represent the amount of f(x) made up by sin (nx)

The **discrete spectrum of f(x)** is given by:

$$\sqrt{(A^2_n + B^2_n)}$$

Essentially, the discrete spectrum of f(x) measures the amount of f(x) with frequency n. In order words, the magnitude of the component in f(x) with frequency n.

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So, magnitude of the component in f(x) with frequency n = 0:
$\sqrt{(A^2_n + B^2_n)} = \sqrt{1} = 1$

Magnitude of the component in f(x) with frequency n = 1:
$\sqrt{(A^2_n + B^2_n)} = \sqrt{(1+1)} = \sqrt{2}$

Magnitude of the component in f(x) with frequency n = 2:
$\sqrt{(A^2_n + B^2_n)} = \sqrt{(1/4 + 0)} = 1/2$

Magnitude of the component in f(x) with frequency n = 3:
$\sqrt{(A^2_n + B^2_n)} = \sqrt{(0 + 1/25)} = 1/5$

Magnitude of the component in f(x) with frequency n = 4:
$\sqrt{(A^2_n + B^2_n)} = \sqrt{(1/16 + 0)} = 1/4$

Magnitude of the component in f(x) with frequency n > 4 is 0.

The String: $S_7P_6A_{61}$ (Grouping/Interaction – Single Criterion Combination).

The Pj Problem of interest is of type *grouping – single criterion combination*. The resolution of f(x) is a decomposition of f(x) into its components. Usually, decomposition problems are of type *change* or *force* (pull) depending on the focus of the problem . However in this problem we are interested in the *grouping* of the components of f(x) according to the *frequency criterion*. It is in this sense that the problem of interest is of type *grouping – single criterion combination*. If the order of the grouping is vital, then the problem of interest will be *grouping – single criterion permutation*.